BIANCHI TYPE-I COSMOLOGICAL MODELS WITH VARYING COSMOLOGICAL TERM-Λ

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ABSTRACT

In this paper, we investigate variation law for cosmological constant in the background of anisotropic Banchi type I space-time. We consider a particular form of cosmological term Λ as Λ = $\beta \ddot{R}/R$, where R is a average scale factor of the universe and β is a constant. The model obtained approaches isotropy at late times. For β = 1 model represent static universe. Physical and kinematical behavior of the model has been discussed.

KEY WORDS: Cosmological models; Bianchi I space-time; Cosmological term; Constant deceleration parameter.

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The present study deals with a spatially homogeneous and anisotropic Bianchi-I cosmological models with decaying cosmological constant Λ. The cosmological constant (Λ) was introduced by Einstein in 1917 as the universal repulsion to make the universe static in accordance with generally accepted picture of that time but Λ can not explain why the calculated value of vacuum energy density at plank epoch is larger than its value as observed by standard cosmology at the present epoch (Weinberg, 1989). In attempt to solve this problem variable Λ was introduced such that Λ was larger in the early universe and then decayed with the evolution (Dolgov, 1983). The idea that Λ might be variable have been studied for more than two decades (Alder, 1982; Weinberg, 1967). Linde (1974) has suggested that Λ is a function of time. A dynamic cosmological cosmological term Λ (t) remains a focal point of interest in modern cosmological theories in a natural way. Liu and Wesson (2001) have studied universe models with variable cosmological constant. The mechanism of the decaying cosmological term is usually formulated in terms of a scalar field (Weinberg, 1972). The generalized scalar tensor theory (Nordtvedt, 1970) has been revived recently and used to solve the cosmological constant problem (Ratra and Peebles, 1988; 2003).

The simplest homogeneous and anisotropic models are Bianchi type-I whose optical sections are that but the expansion or contraction rate is directional dependent. The advantages of these anisotropic models are that they have a significant role in the description of the evolution of the early phase of the universe and they help in finding move general cosmological models than the isotropic Friedman- Robertson Walker models. The isotropy of the present day universe makes the Bianchi-I model a prime for studying the possible effects of an anisotropy in the early universe on modern day data observations.

In this paper we study Bianchi type-I model with decaying cosmological constants. This work is organized as follows. The model and field evaluation are given in section-2. The field equation are solved in section-3. The physical behavior of the model is discussed in detail is last section.

Metric and field equation :

We consider the Bianchi type-I space time described by the line-element

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2 \quad \ldots (1)$$

We assume the cosmic matter consisting of perfect fluid represented by the energy-momentum tensor

$$T_{ij} = (\rho + p) v_i v_j + p g_{ij} \qquad \ldots (2)$$

satisfying linear equation of state

$$p = w\rho , \qquad 0 \le w \le 1 \qquad \ldots (3)$$

Where ρ and p are energy density and isotropic pressure respectively and v^i the four velocity vector of the fluid satisfying $v_i v^i = -1$. The Einstein's field equations (in gravitational units 8πG = c = 1) with time-dependent cosmological term Λ (t) are

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j + \Lambda g_i^j \qquad .. (4)$$

For the line-element (1), the above field equations in comoving system of coordinates lead to

$$p - \Lambda = -\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} \qquad \ldots (5)$$

$$p - \Lambda = -\frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} - \frac{\dot{A}\dot{C}}{Ac} \qquad \ldots (6)$$

$$p - \Lambda = -\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} \qquad \ldots (7)$$

$$\rho + \Lambda = \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} \qquad \ldots (8)$$

In the above and elsewhere an overhead dot (.) stands for ordinary time-derivative of the concerned quantity. Covariant divergence of (4) leads to

$$\dot{\rho} + (\rho + p)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) + \dot{\Lambda} = 0 \qquad \ldots (9)$$

We define average scale factor R as R^3 = ABC and introduce volume expansion θ and shear scalar as usual

$$\theta = v^i_{;i} , \qquad \sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \qquad \ldots (10)$$

σ_{ij} being shear tensor. In the above semicolon stands for covariant differentiation. From (5) – (7) in view of R^3 = ABC we obtain

$$\frac{\dot{A}}{A}=\frac{\dot{R}}{R}+\frac{2k_1+k_2}{3R^3} \quad \ldots(11)$$

$$\frac{\dot{B}}{B}=\frac{\dot{R}}{R}+\frac{k_2-k_1}{3R^3} \quad \ldots(12)$$

$$\frac{\dot{C}}{C}=\frac{\dot{R}}{R}-\frac{k_1+2k_2}{3R^3} \quad \ldots(13)$$

where k_1 and k_2 are integration constants.

For the Bianchi type-I metric expressions of (10) come not to be

$$\theta=\frac{3\dot{R}}{R} \quad \ldots (14)$$

$$\sigma=\frac{k}{R^3} \quad \ldots (15)$$

Where $3k^2=k_1^2+k_2^2+k_1k_2$. In analogy with the FRW universe, we define a generalized Hubble parameter H and the generalized deceleration parameter q as

$$H=\dot{R}/R \quad \ldots (16)$$

$$q=-\frac{\ddot{R}}{RH^2} \quad \ldots (17)$$

Equations (5) – (8) can be expressed in terms of H, σ and q as

$$p-\Lambda=(2q-1)H^2-\sigma^2 \quad \ldots (18)$$

$$\rho+\Lambda=3H^2-\sigma^2 \quad \ldots (19)$$

Solution of the field equations :

From equations (3), (18) and (19) we obtain

$$\frac{1}{2}(1-w)\,\rho+\Lambda=\frac{\ddot{R}}{R}+\frac{2\dot{R}^2}{R^2}-\frac{2}{R^2} \qquad \text{... (20)}$$

Thus, we have one equation with three unknowns R, ρ and Λ. We require two more relations to close the system. We assume that w = 1 (stiff fluid) and

$$\Lambda=\beta\frac{\ddot{R}}{R} \qquad \text{... (21)}$$

where β is a constant. Therefore, equation (20) becomes

$$(\beta-1)\frac{\ddot{R}}{R}-\frac{2\dot{R}^2}{R^2}=0 \qquad \text{... (22)}$$

Integrating (22), we get

$$R=a(b+t)^{\frac{\beta-1}{\beta-3}} \qquad \text{... (23)}$$

where a and b are integration constant.

For this solution metric (1) assumes the following form

$$ds^2=-dt^2+a^2(b+t)^{\frac{2(\beta-1)}{\beta-3}}$$

$$\left[e^{\frac{(3-\beta)(2k_1+k_2)(b+t)^{2\beta/(3-\beta)}}{3a^3\beta}}dx^2\right.$$

$$+e^{\frac{(3-\beta)(k_2-k_1)(b+t)^{2\beta/(3-\beta)}}{3a^3\beta}}dy^2$$

$$\left.+e^{\frac{-(3-\beta)(k_1+2k_2)(b+t)^{2\beta/(3-\beta)}}{3a^3\beta}}dz^2\right] \qquad \text{... (24)}$$

Discussion :

Matter density ρ and cosmological term Λ for the model (24) are given by

$$\rho = \frac{\beta - 1}{(\beta - 3)(b+t)^2} - \frac{k^2}{a^6}(b+t)^{\frac{6(\beta-1)}{\beta-3}} \qquad \ldots(25)$$

$$\Lambda = \frac{2\beta(\beta - 1)}{(\beta-3)^2 (b+t)^2} \qquad \ldots(26)$$

Average scale factor R, Hubble parameter H, deceleration parameter q, shear scalar σ for the model come out to be

$$R = a(b+t)^{\frac{\beta-1}{\beta-3}}$$

$$H = \frac{(\beta-1)}{(\beta-3)(b+t)}$$

$$q = -\left(\frac{2}{\beta-1}\right)$$

$$\sigma = \frac{k}{a^3 (b+t)^{3(\beta-1)/(\beta-3)}}$$

We observe that β > 1 and β < 3 i.e. 1 < β < 3. For β = 1, R = a, therefore model represent static universe. The model has initial singularity at t = -b. The model starts with a big bang at t = -b with ρ, Λ, θ, σ all infinite and q < 0. In the limit t → ∞, ρ, Λ, θ, σ all become zero. In this model, we have a accelerating expansion throughout. Also $\lim_{t\to\infty} \sigma/\theta = 0$. Thus, model approaches isotropy at late times.

References :

- Weinberg, S. 1989, The cosmological constant problem, *Rev. Mod. Phys.* **61**, 1.
- Dolgov, A.D. 1983. In The Very Early Universe, eds. By G.W. Gibbons, S.W. Hawking and S.T.C. Siklos, Cambridge University Press, Cambridge, p.449.
- Alder, S.L. 1982, Einstein gravity as a symmetry-breaking effect in quantum field theory, *Rev. mod. Phys.* **54**, 729.
- Weinberg, S. 1967, A Model of Leptons, *Phys. Rev. Let.*, **19**, 1264.
- Linde, A.D. 1974, Is the Lee constant a cosmological constant?, *Sov. Phys. Lett.* **19**, 183.
- Weinberg, S. 1972, *Gravitation and Cosmology*. (Wiley, New York).
- Fujii, Y. and Nishioka, T. 1990, Model of a decaying cosmological constant, *Phys. Rev.* **D42**, 361.
- Barrow, J.D., Maeda, K. 1990, Extended inflationary universes, *Nucl. Phys.* **B341**, 294.
- Nordtvedt, K. 1970, Post-Newtonian Metric for a General Class of Scalar-Tensor Gravitational Theories and Observational Consequences, *Ap. J.*, **161**, 1059.
- Ratra, B.. Peebles, P.J.E. 2003, The cosmological constant and dark energy, *Rev. Mod. Phys.*, **75**, 559.
- Ratra, B., Peebles, P.J.E. 1998, Cosmological consequences of a rolling homogeneous scalar field, *Phys. Rev.* **D37**, 3406.